

# GRAVITATIONAL COLLAPSE IN VAIDYA SPACE-TIME FOR GALILEON GRAVITY THEORY

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## Abstract

The motive of this work is to study gravitational collapse in Vaidya space-time embedded in Galileon gravity theory. Galileon gravity is in fact an infrared modification of Einstein gravity, which was proposed as a generalization of the 4D effective theory in DGP brane model. Vaidya's metric is used all over to follow the nature of future outgoing radial null geodesics. Detecting whether the central singularity is naked or wrapped by an event horizon, by the existence of future directed radial null geodesic emitted in past from the singularity is the basic objective. To point out the existence of positive trajectory tangent solution, both particular parametric cases (through tabular forms) and wide range contouring process have been applied. Precisely, the EoS in perfect fluid satisfies a wide range of phenomena: from dust to exotic fluid like dark energy. We have used the EoS parameter  $k$  to determine the end collapse state in different cosmological era.

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## 1 INTRODUCTION

Right from the discovery of Newton's theory of gravitation, one question that has been haunting the human mind is 'how gravity works at cosmological distances?' Although there was no definite answer to this question, yet there was a feeling that the story at large distances is quite different to Newton's gravitation. With the discovery of the cosmic acceleration in the late twentieth century, this feeling started transforming into belief. As a possible explanation for the cosmic acceleration, modifications to gravity at cosmological distances have recently received much attention. Numerous modified gravity theories have been proposed during the last decade, some of which worth mentioning are, loop Quantum gravity (LQG) [1, 2], Gauss-Bonnet gravity [3, 4], scalar-tensor theories [5, 6, 7],  $f(R)$  gravity [8], DGP braneworld model [9], Galileon gravity [10, 11, 12, 13, 14], etc.

Modification to gravity can be done keeping in mind that the deviations from General Relativity (GR) can only be allowed at large distances. At close distances the theory should coincide with Newton's theory of gravitation thus ensuring the consistency of the model with the solar system experiments. We know that the DGP brane model admits a self-accelerating solution (cosmic acceleration in the absence of any matter in the universe) [15]. But unfortunately this solution is plagued with ghost instabilities, which renders the model rather invalid [16]. There are many other modified gravity models suffering from this instability problem [17].

Of late an infrared modification of classical gravitation was proposed, as a generalization of the 4D effective theory in the DGP model [10]. The theory considers a self-interaction term of the form  $(\nabla\phi)^2 \square \phi$  in order

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to recover GR in high density regions. The most striking feature of the theory is that it is invariant under the Galileon shift symmetry,  $\delta_\mu\phi \rightarrow \delta_\mu\phi + c_\mu$ , in the Minkowski background. Due to this invariance the equation of motion remains a second order differential equation, preventing the introduction of extra degrees of freedom, which are usually associated with instabilities.

The study of gravitational collapse was started by Oppenheimer and Snyder in 1939 [18]. In classical GR, the gravitational collapse is a problem of great curiosity as we can get at least two types of singularities from it. One, covered by an event horizon, is coined as a black hole (BH) whereas the singularity alone is popular as Naked Singularity (NS). Now, one would always like to test the validity of cosmic censorship hypothesis (CCH) laid down by Penrose, [19] which stated that the end result of a collapse is bound to be a singularity covered by an event horizon, i.e., BH. In last few decades there have been extensive research [20, 21, 22, 23, 24, 25] where the possibility of formation of NS has been investigated. To determine the end state of collapse (NS or BH), the Vaidya solution [26] is utilized on many occasions. Harko et al [27] have also studied the gravitational collapse in Vaidya space-time.

Gravitational collapse has been extensively studied in modified gravity theory i.e., in Gauss-Bonnet,  $f(R)$  gravity, Lovelock theory etc [28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46]. Scheel [47] demonstrated that Openheimer-Snyder collapse in Brans-Dicke theory results in BHs rather than NSs with the positive values of brans-dicke parameter  $\omega$ , they have speculated that the apparent horizon of a BH can pass outside the event horizon causing the decrease in the surface area over time. The values of  $\omega$  forces the BH, to radiate its scalar mass to infinity soon after the initial collapse. In [48] we get another relevant and interesting work regarding the gravitational collapse in the background of Brans-Dicke theory of gravity discussing the effects of different values of  $\omega$ . Rudra et al in [49] has shown that lower the value of  $\omega$ , greater is the chance of getting an NS.

In 1996, Husain [50] has studied the non-static spherically symmetric solutions of Einstein equations, for a null fluid source, where the density  $\rho$  and pressure  $p$  of the fluid is related by barotropic equation of state  $p = k\rho$ . All of the new solutions supported the cosmic censorship conjecture. Later the Vaidya solution was generalized by Wang et al [51]. Moreover the Husain solution has been extensively used to study the formation of a black hole with short hair [52]. The most recent development in Husain solution was witnessed when the gravitational collapse of the Husain solution in four and five dimensional space-times was studied by Patil et al [39].

Keeping all the previous works of gravitational collapse in GR/ different gravity theories in mind we feel it will be of a great interest if we investigate the existence of radial null geodesic from the collapsing body in the back ground of Galileon gravity theory. The scalar factor present in the theory may help in collapse to form a NS more prominently than the GR does. As in [49] we will check that whether the nature of the end state of collapse can be regulated by any parameter of Galileon gravity. In this concern we must recall the fact that in [44] while working with Lovelock gravity, we saw that greater the deviation from Einstein gravity greater was the tendency to have the NS. So in this paper, we are mainly studying the nature of singularities (BH or NS) formed by the gravitational collapse in Galileon gravity and the conditions governing these. In section (2), we present the brief overview of generalized Vaidya solution in Galileon gravity. We will first construct the Einstein field equations in Galileon theory for the Vaidya metric and then with a proper choice of the structural dependence of the potential term upon the scalar field we will determine the  $m(t, r)$ , the mass term. In the next two sections we investigate the behaviour/existence of the outgoing radial null geodesic from the singularity taking the Vaidya metric with the mass term  $m(t, r)$  derived in the last section. Finally, the paper ends with some concluding remarks in section (5).

## 2 Field Equations and the Solutions in Vaidya Space-Time in the background of Galileon Gravity

The Galileon theory is described by the action [10, 11, 12, 13, 14]:

$$S = \int d^4x \sqrt{-g} \left[ \phi R - \frac{\omega}{\phi} (\nabla\phi)^2 + f(\phi) \square \phi (\nabla\phi)^2 + \mathcal{L}_m \right] \quad (1)$$

where  $\phi$  is the Galileon field and the coupling  $f(\phi)$  has dimension of length,  $(\nabla\phi)^2 = g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$ ,  $\square\phi = g^{\mu\nu}\nabla_\mu\nabla_\nu\phi$  and  $\mathcal{L}_m$  is the matter Lagrangian. Here we consider the metric in spherically symmetric space-time in the form [26]

$$ds^2 = -\left(1 - \frac{m(t, r)}{r}\right) dt^2 + 2dt dr + r^2 d\Omega_2^2 \quad (2)$$

where  $r$  is the radial co-ordinate and  $t$  is the null co-ordinate,  $m(t, r)$  gives the gravitational mass inside the sphere of radius  $r$  and  $d\Omega_2^2$  is the line element on a unit 2-sphere.

Variation with respect to the metric gives the Einstein's equations,

$$G_{\mu\nu} = \frac{T_{\mu\nu}}{2\phi} + \frac{1}{\phi} (\nabla_\mu\nabla_\nu\phi - g_{\mu\nu}\square\phi) + \frac{\omega}{\phi^2} \left[ \nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}g_{\mu\nu}(\nabla\phi)^2 \right] - \frac{1}{\phi} \left\{ \frac{1}{2}g_{\mu\nu}\nabla_\lambda[f(\phi)(\nabla\phi)^2]\nabla^\lambda\phi - \nabla_\mu[f(\phi)(\nabla\phi)^2]\nabla_\nu\phi + f(\phi)\nabla_\mu\phi\nabla_\nu\phi\square\phi \right\} \quad (3)$$

Now we consider two types of fluids like Vaidya null radiation and a perfect fluid having the form of the energy-momentum tensor [44]

$$T_{\mu\nu} = T_{\mu\nu}^{(n)} + T_{\mu\nu}^{(m)} \quad (4)$$

with

$$T_{\mu\nu}^{(n)} = \sigma l_\mu l_\nu \quad (5)$$

and

$$T_{\mu\nu}^{(m)} = (\rho + p)(l_\mu\eta_\nu + l_\nu\eta_\mu) + pg_{\mu\nu} \quad (6)$$

where,  $\rho$  and  $p$  are the energy density and pressure for the perfect fluid and  $\sigma$  is the energy density corresponding to Vaidya null radiation. The two eigen vectors of energy-momentum tensor namely  $l_\mu$  and  $\eta_\mu$  are linearly independent future pointing null vectors having components

$$l_\mu = (1, 0, 0, 0) \quad \text{and} \quad \eta_\mu = \left(\frac{1}{2}\left(1 - \frac{m}{r}\right), -1, 0, 0\right) \quad (7)$$

and they satisfy the relations

$$l_\lambda l^\lambda = \eta_\lambda \eta^\lambda = 0, \quad l_\lambda \eta^\lambda = -1 \quad (8)$$

The Einstein field equations ( $G_{\mu\nu} = T_{\mu\nu}$ ) for the metric (2) and the wave equation for the Galileon field  $\phi$  are the following:

**From  $G_{00} = T_{00}$  we get,**

$$\begin{aligned} \frac{(r-m)m' + r\dot{m}}{r^3} &= \frac{\sigma + \rho\left(1 - \frac{m}{r}\right)}{2\phi} + \frac{1}{\phi} \left[ \ddot{\phi} - \left(\frac{m}{2r^2} - \frac{m'}{2r}\right)\dot{\phi} - \left(\frac{m}{2r^2} - \frac{m^2}{2r^3} - \frac{m'}{2r} + \frac{mm'}{2r^2} + \frac{\dot{m}}{2r}\right)\phi' \right. \\ &+ \left. \left(1 - \frac{m}{r}\right) \left\{ 2\dot{\phi}' - \phi' \left(\frac{m'}{r} - \frac{3m}{r^2} + \frac{2}{r}\right) + \left(1 - \frac{m}{r}\right)\phi'' \right\} \right] + \frac{\omega}{\phi^2} \left[ \dot{\phi}^2 + \frac{1}{2}\left(1 - \frac{m}{r}\right)\phi' \left(2\dot{\phi} + \left(1 - \frac{m}{r}\right)\phi'\right) \right] \\ &+ \frac{1}{\phi} \left[ \frac{1}{2}\left(1 - \frac{m}{r}\right) \left\{ \phi'\nabla_0 U + \left(\dot{\phi} + \left(1 - \frac{m}{r}\right)\phi'\right)\nabla_1 U \right\} - \dot{\phi}\nabla_0 U + f(\phi)\dot{\phi}^2 \left\{ 2\dot{\phi}' - \phi' \left(\frac{m'}{r} - \frac{3m}{r^2} + \frac{2}{r}\right) \left(1 - \frac{m}{r}\right)\phi'' \right\} \right], \end{aligned} \quad (9)$$

**$G_{11} = T_{11}$  gives,**

$$\begin{aligned} \frac{\phi''}{\phi} + \frac{\omega\phi'^2}{\phi^2} - \frac{1}{\phi} \left[ -f(\phi) \left\{ 2\phi'^2\dot{\phi}' + 2\dot{\phi}\phi'\phi'' + 2\left(1 - \frac{m}{r}\right)\phi'^2\phi'' + \frac{m}{r^2}\phi'^3 \right\} - f'(\phi) \left\{ 2\phi'^3\dot{\phi} + \left(1 - \frac{m}{r}\right)\phi'^4 \right\} \right. \\ \left. + f(\phi)\phi'^2 \left\{ 2\dot{\phi}' - \phi' \left(\frac{m'}{r} - \frac{3m}{r^2} + \frac{2}{r}\right) + \left(1 - \frac{m}{r}\right)\phi'' \right\} \right] = 0, \end{aligned} \quad (10)$$

$G_{01} = T_{01}$  gives,

$$\begin{aligned} \frac{m'}{r^2} &= \frac{\rho}{2\phi} + \frac{1}{\phi} \left[ \dot{\phi}' + \phi' \left( \frac{m'}{2r} - \frac{m}{2r^2} \right) - \phi' \left( \frac{m'}{r} - \frac{3m}{r^2} + \frac{2}{r} \right) + \phi'' \left( 1 - \frac{m}{r} \right) \right] + \frac{\omega}{2\phi^2} \left( 1 - \frac{m}{r} \right) \phi'^2 \\ &+ \frac{1}{\phi} \left[ \frac{1}{2} \nabla_0 U \left( \dot{\phi} - 2\phi' \right) + \frac{1}{2} \phi' \nabla_1 U + f(\phi) \dot{\phi} \phi' \left\{ 2\dot{\phi}' - \phi' \left( \frac{m'}{r} - \frac{3m}{r^2} + \frac{2}{r} \right) + \left( 1 - \frac{m}{r} \right) \phi'' \right\} \right], \end{aligned} \quad (11)$$

$G_{22} = T_{22}$  gives,

$$\begin{aligned} \frac{1}{2} r m'' &= \frac{\omega}{\phi^2} \left[ \frac{r^2}{2} \phi' \left\{ 2\dot{\phi} + \left( 1 - \frac{m}{r} \right) \phi' \right\} \right] - \frac{1}{\phi} \left[ r^2 \left\{ \phi' \left( \frac{m'}{r} - \frac{3m}{r^2} + \frac{2}{r} \right) - \left( 1 - \frac{m}{r} \right) \phi'' - 2\dot{\phi}' \right\} \right] \\ &+ \frac{r^2}{2\phi} \left( \nabla_0 U \dot{\phi} + \nabla_1 U \phi' \right) - \frac{p r^2}{2\phi} \end{aligned} \quad (12)$$

and

$G_{33} = T_{33}$  gives,

$$\begin{aligned} \frac{p r^2}{2\phi} + \frac{1}{\phi} \left[ r \dot{\phi} - (m - r) \phi' - r^2 \left\{ 2\dot{\phi}' - \phi' \left( \frac{m'}{r} - \frac{3m}{r^2} + \frac{2}{r} \right) + \left( 1 - \frac{m}{r} \right) \phi'' \right\} \right] - \frac{\omega}{\phi^2} \left[ \frac{\phi'}{2} r^2 \left( 2\dot{\phi} + \phi' \left( 1 - \frac{m}{r} \right) \right) \right] \\ - \frac{1}{2\phi} r^2 \left( \dot{\phi} \nabla_0 U + \phi' \nabla_1 U \right) + \frac{1}{2} r m'' = 0 \end{aligned} \quad (13)$$

where an over-dot and dash stand for differentiation with respect to  $t$  and  $r$  respectively. Here  $U = f(\phi) (\nabla \phi)^2$  and correspondingly the expressions for  $\nabla_0 U$  and  $\nabla_1 U$  are given as below:

$$\nabla_0 U = f(\phi) \left[ 2\phi' \ddot{\phi} + 2\dot{\phi} \dot{\phi}' + \left( 1 - \frac{m}{r} \right) 2\phi' \dot{\phi}' - \phi'^2 \frac{\dot{m}}{r} \right] + f'(\phi) \left[ 2\phi' \dot{\phi}^2 + \left( 1 - \frac{m}{r} \right) \phi'^2 \dot{\phi} \right], \quad (14)$$

$$\nabla_1 U = f(\phi) \left[ 2\phi' \dot{\phi}' + 2\dot{\phi} \phi'' + 2 \left( 1 - \frac{m}{r} \right) \phi' \phi'' + \frac{m}{r^2} \phi'^2 \right] + f'(\phi) \left[ 2\phi'^2 \dot{\phi} + \left( 1 - \frac{m}{r} \right) \phi'^3 \right] \quad (15)$$

Due to the complicated nature of the field equations we cannot solve them directly and get the expression for  $\phi$ . Therefore we assume

$$\phi(r, t) = P(r)Q(t) \quad (16)$$

where  $P(r)$  is an arbitrary function of  $r$  and  $Q(t)$  is an arbitrary function of  $t$ . Since  $f(\phi)$  is an arbitrary function of  $\phi$ , **so in order to facilitate calculations, we choose,**

$$f(\phi) = f_0 \phi^{-2} \quad (17)$$

where,  $f_0$  is a constant. We assume the matter fluid obeys the barotropic equation of state

$$p = k\rho, \quad (k, a \text{ constant}) \quad (18)$$

Using equations (11), (12), (14), (15), (16) and (17) we have the solution for  $Q(t)$  as,

$$Q(t) = \alpha_1 e^{-\lambda t} \quad (19)$$

where  $\alpha_1$  and  $\lambda$  are arbitrary constants. Since no solution for  $P(r)$  could be obtained due to the highly complicated nature of the field equations, we consider,

$$P(r) = \alpha r^n \quad (20)$$

where  $\alpha$  and  $n$  are arbitrary constants. Now using the above values of  $P$  and  $Q$  in the field equations we finally arrive at a differential equation in  $m(t, r)$  as,

$$r^2 m'' + [k + n(2 + k)] r m' + [n \{2(k + 1)(n - 1) - (5k + 6)\}] m + 2n [(3 - n)(k + 1)r + (\omega + k + 2)\lambda r^2] = 0 \quad (21)$$

Solving the above differential equation we obtain the explicit solution for  $m$  as,

$$m(t, r) = f_1(t)r^{\omega_1} + f_2(t)r^{\omega_2} + \frac{2n(3 - n)(k + 1)}{(1 - \omega_1)(1 - \omega_2)}r + \frac{2n\lambda(\omega + k + 2)}{(2 - \omega_1)(2 - \omega_2)}r^2 \quad (22)$$

where

$$\omega_1, \omega_2 = \frac{[1 - k - n(2 + k)] \pm \sqrt{\{k + n(2 + k) - 1\}^2 - 4n\{2(k + 1)(n - 1) - (5k + 6)\}}}{2} \quad (23)$$

Here  $f_1(t)$  and  $f_2(t)$  are arbitrary functions of  $t$ .

Therefore the metric (2) can be written as

$$ds^2 = \left[ -1 + f_1(t)r^{\omega_1-1} + f_2(t)r^{\omega_2-1} + \frac{2n(3 - n)(k + 1)}{(1 - \omega_1)(1 - \omega_2)} + \frac{2n\lambda(\omega + k + 2)}{(2 - \omega_1)(2 - \omega_2)}r \right] dt^2 + 2dt dr + r^2 d\Omega_2^2 \quad (24)$$

which is called the Generalized Vaidya metric in Galileon gravity.

### 3 Collapse Study

We shall discuss the existence of NS in generalized Vaidya space-time by studying radial null geodesics. In fact, we shall examine whether it is possible to have outgoing radial null geodesics which were terminated in the past at the central singularity  $r = 0$ . The nature of the singularity (NS or BH) can be characterized by the existence of radial null geodesics emerging from the singularity. The singularity is at least locally naked if there exist such geodesics and if no such geodesics exist it is a BH.

Let  $R(t, r)$  is the physical radius at time  $t$  of the shell labelled by  $r$ . At the starting epoch  $t = 0$  we have assumed the scaling (freedom)  $R(0, r) = r$ . Now if there are future directed radial null geodesics coming out of the singularity, with a well defined tangent at the singularity  $\frac{dR}{dr}$  must tend to a finite limit in the limit of approach to the singularity in the past along these trajectories.

The point  $(t_0, r) = (0, 0)$  occurs, where the singularity  $R(t_0, 0) = 0$  occurs corresponds to the physical situation where matter shells are crushed to zero radius. This kind of singularity ( $r = 0$ ) is known to be a central shell focusing singularity. The singularity is a NS if there are future directed non-space like curves in the space time with their past end points at the singularity. Now if the outgoing radial null geodesics are to terminate in the past at the central singularity at  $r = 0$  at  $t = t_0$  where  $R(t_0, 0) = 0$ , then along these geodesics we should have [33]  $R \rightarrow 0$  as  $r \rightarrow 0$ .

The equation for outgoing radial null geodesics can be obtained from equation (2) by putting  $ds^2 = 0$  and  $d\Omega_2^2 = 0$  as

$$\frac{dt}{dr} = \frac{2}{\left(1 - \frac{m(t, r)}{r}\right)}. \quad (25)$$

Now we consider the time  $t = 0$ , when the singularity forms at the centre  $r = 0$ . So we can recalled that  $r = 0, t = 0$  corresponds to a central singularity. It can be seen easily that  $r = 0, t = 0$  corresponds to a singularity of the above differential equation. Suppose  $X = \frac{t}{r}$  then we shall study the limiting behavior of the function  $X$  as we approach the singularity at  $r = 0, t = 0$  along the radial null geodesic. If we denote the limiting value by  $X_0$  then

$$X_0 = \lim_{\substack{t \rightarrow 0 \\ r \rightarrow 0}} X = \lim_{\substack{t \rightarrow 0 \\ r \rightarrow 0}} \frac{t}{r} = \lim_{\substack{t \rightarrow 0 \\ r \rightarrow 0}} \frac{\frac{dt}{dr}}{\frac{dr}{dt}} = \lim_{\substack{t \rightarrow 0 \\ r \rightarrow 0}} \frac{2}{\left(1 - \frac{m(t, r)}{r}\right)} \quad (26)$$

Using equations (22) and (26), we have

$$\frac{2}{X_0} = \lim_{\substack{t \rightarrow 0 \\ r \rightarrow 0}} \left[ 1 - f_1(t)r^{\omega_1-1} - f_2(t)r^{\omega_2-1} - \frac{2n(3 - n)(k + 1)}{(1 - \omega_1)(1 - \omega_2)} - \frac{2n\lambda(\omega + k + 2)}{(2 - \omega_1)(2 - \omega_2)} \frac{r}{t} \right] \quad (27)$$

Now choosing  $f_1(t) = \delta t^{-(\omega_1-1)}$  and  $f_2(t) = \epsilon t^{-(\omega_2-1)}$ , ( $\delta$  and  $\epsilon$  are constants), we obtain the algebraic equation of  $X_0$  as

$$\delta X_0^{2-\omega_1} + \epsilon X_0^{2-\omega_2} - \left[ 1 - \frac{2n(3-n)(k+1)}{(1-\omega_1)(1-\omega_2)} \right] X_0 + 2 \left[ 1 + \frac{n\lambda(\omega+k+2)}{(2-\omega_1)(2-\omega_2)} \right] = 0 \quad (28)$$

Outgoing radial null geodesic exists if the value of the tangent near the singularity is positive i.e.,  $X_0 > 0$ . Now if we get only non-positive solution of the equation we can assure the formation of a BH. Getting a positive root indicates a chance to get a NS. Since the obtained equation is a highly complicated one, it is extremely difficult to find out an analytic solution of  $X_0$  in terms of the variables involved. So our idea is to find out different numerical solutions of  $X_0$ , by assigning particular numerical values to the associated constants.

The different solutions of  $X_0$  for different sets of parametric values of  $(\omega, \lambda, \delta, \epsilon, n)$  and for different stages of EoS parameter  $k$ , which are given here in a tabular form (Table 1a-e).

For $k = 1$ (stiff perfect fluid)					
$\omega$	$\lambda$	$\delta$	$\epsilon$	$n$	Positive roots ( $X_0$ )
3	1	1	1	1	—
3	1	1	1	2	—
3	1	1	1	4	0.751311
2	3	0.1	0.2	1	—
2	3	0.1	0.2	2	0.864566
2	3	0.1	0.2	4	0.911302
1	0.1	2	1	1	—
1	0.1	2	1	2	—
1	0.1	2	1	4	0.690284
-2	0.1	0.1	0.1	1	—
2	0.1	0.1	0.1	2	0.91185
2	0.1	0.1	0.1	4	0.937988
-3	0.1	0.1	1	1	—
3	0.1	0.1	1	2	0.761759
3	0.1	0.1	1	4	0.846213
-4	0.1	0.1	1	1	—
-4	0.1	0.1	1	2	0.761758
-4	0.1	0.1	1	4	0.846213

Table1a

Table1b	For $k = 1/3$ (radiation)					
	$\omega$	$\lambda$	$\delta$	$\epsilon$	$n$	Positive roots ( $X_0$ )
	2	0.1	0.1	1	1	1.24846
	2	0.1	0.1	1	2	0.688704
	2	0.1	0.1	1	4	0.818295
	1	0.1	2	0.1	1	0.691721
	1	0.1	2	0.1	2	0.178112
	1	0.1	2	0.1	4	0.641779
	2	1	0.1	1	1	1.25072
	2	1	0.1	1	2	0.688701
	2	1	0.1	1	4	0.818295
	-3	0.1	0.1	3	1	1.0127
	3	0.1	0.1	3	2	0.610956
	3	0.1	0.1	3	4	0.759971
	-4	0.1	0.1	1	1	1.261581
	-4	0.1	0.1	1	2	0.688701
	-4	0.1	0.1	1	4	0.818295

Table1c	For $k = -0.5$ (dark energy)					
	$\omega$	$\lambda$	$\delta$	$\epsilon$	$n$	Positive roots ( $X_0$ )
	2	0.1	0.1	1	1	0.763465
	2	0.1	0.1	1	2	—
	2	0.1	0.1	1	4	0.761803
	1	0.1	2	1	1	0.7107
	1	0.1	2	1	2	—
	1	0.1	2	1	4	0.510191
	-0.5	3	0.1	0.1	1	1.70536
	0.5	3	0.1	0.1	2	—
	0.5	3	0.1	0.1	4	0.984339
	-2	0.1	0.1	3	1	0.520046
	-2	0.1	0.1	3	2	—
	-2	0.1	0.1	3	4	0.670155
	-3	0.1	0.1	1	1	0.763465
	-3	0.1	0.1	1	2	—
	-3	0.1	0.1	1	4	0.761803
	-5	0.1	0.1	1	1	1.04294
	-5	0.1	0.1	1	2	—
	-5	0.1	0.1	1	4	0.761803

Table1d	For $k = -1$ ( $\Lambda$ CDM)					
	$\omega$	$\lambda$	$\delta$	$\epsilon$	$n$	Positive roots ( $X_0$ )
	3	0.1	0.2	1	1	1.56222
	3	0.1	0.2	1	2	—
	3	0.1	0.2	1	4	0.665843
	1	0.1	0.1	1	1	0.522207
	1	0.1	0.1	1	2	—
	1	0.1	0.1	1	4	0.686147
	-0.5	1	0.1	3	1	0.981042
	-0.5	1	0.1	3	2	0.367083
	-0.5	1	0.1	3	4	0.558615
	-2	0.1	0.1	1	1	0.522208
	-2	0.1	0.1	1	2	0.582659
	-2	0.1	0.1	1	4	0.686145
	-4	0.1	0.1	1	1	0.522216
-4	0.1	0.1	1	2	0.582659	
-4	0.1	0.1	1	4	0.686147	

Table1e	For $k = -2$ (phantom)					
	$\omega$	$\lambda$	$\delta$	$\epsilon$	$n$	Positive roots ( $X_0$ )
	2	0.1	0.1	1	1	—
	2	0.1	0.1	1	2	—
	2	0.1	0.1	1	4	1.11744
	1	0.1	0.1	1	1	—
	1	0.1	0.1	1	2	—
	1	0.1	0.1	1	4	1.11744
	-0.5	0.1	0.1	1	1	—
	-0.5	0.1	0.1	1	2	—
	-0.5	0.1	0.1	1	4	1.11744
	-2	0.1	0.1	1	1	—
	2	0.1	0.1	1	2	—
	2	0.1	0.1	1	4	1.11744
	-3	0.1	2	1	1	—
	3	0.1	2	1	2	—
	3	0.1	2	1	4	1.45782
	-4	0.1	0.1	1	1	—
4	0.1	0.1	1	2	—	
4	0.1	0.1	1	4	1.11745	

**Table 1a-e:** Values of  $X_0$  for different values of parameters  $\lambda, \delta, \epsilon, n, \omega$  and  $k$ .



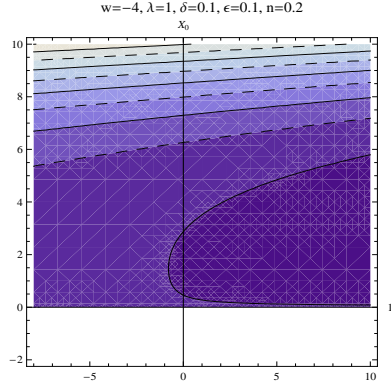


Fig.1a

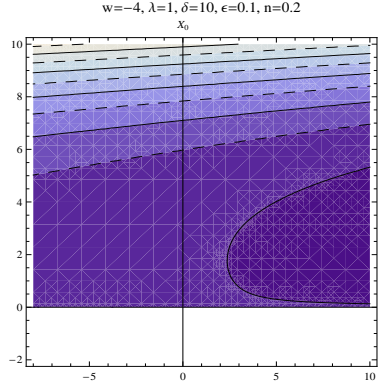


Fig.1b

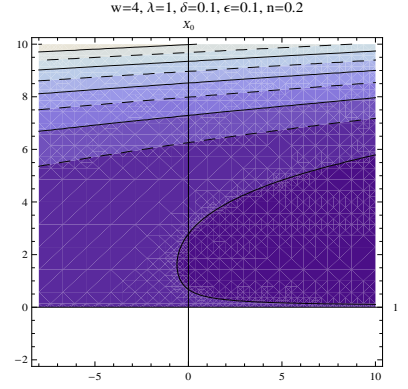


Fig.1c

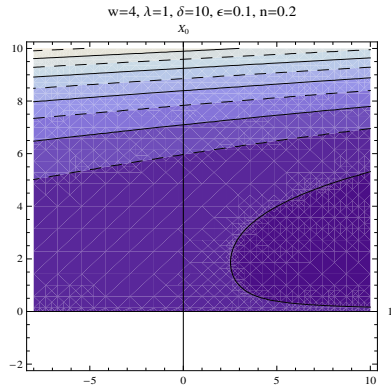


Fig.1d

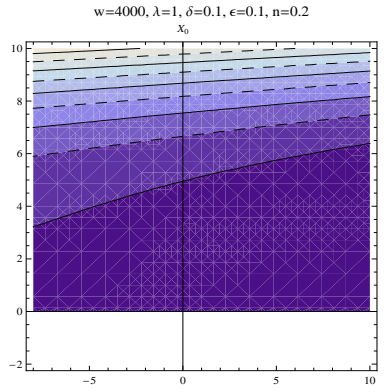


Fig.1e

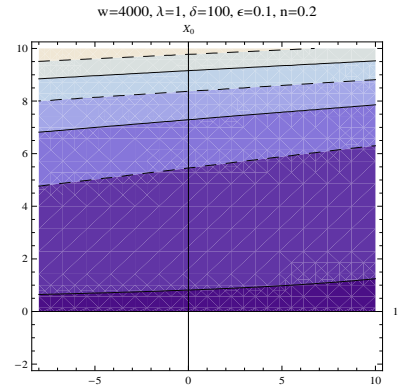


Fig.1f

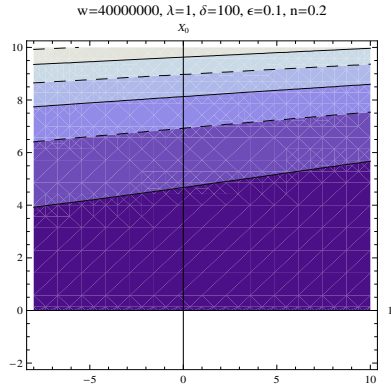


Fig.1g

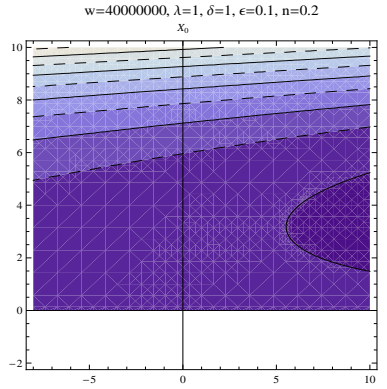


Fig.1h

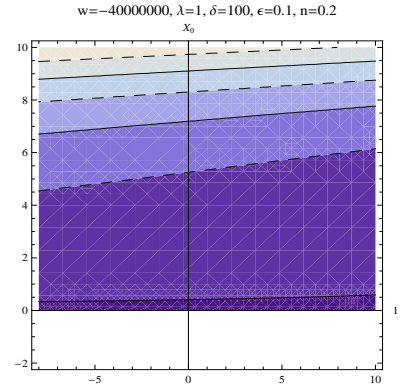


Fig.1i

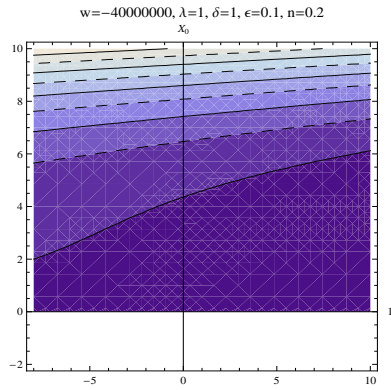


Fig.1j

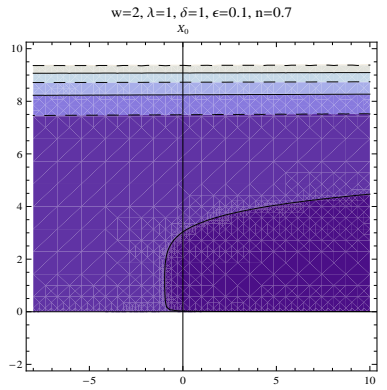


Fig.1k

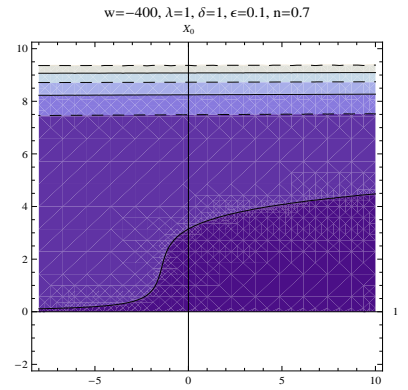


Fig.1l

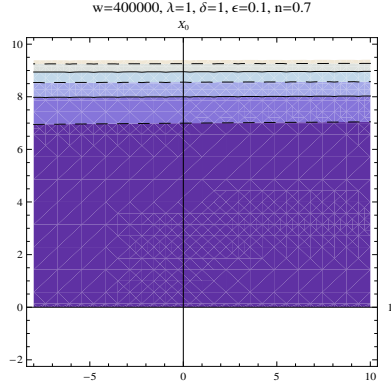


Fig.1m

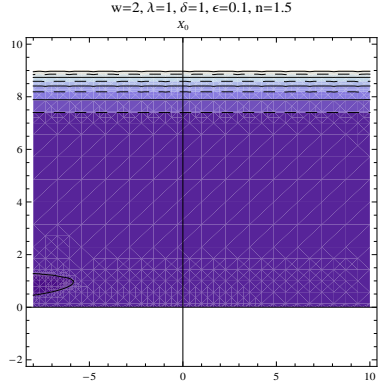


Fig.1n

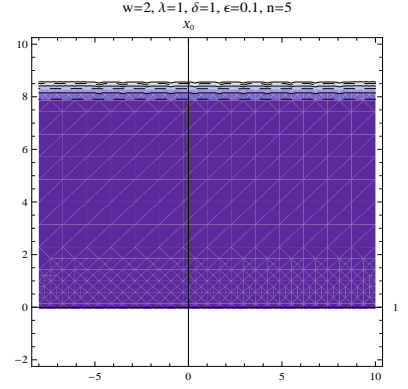


Fig.1o

**Figs. 1a-o** show the  $X_0$  contours in the  $k$ - $X_0$  plane for different values of  $\omega$ .

Looking at the tables, we can see that the result of collapse does not quite depend on the value of the Galileon parameter  $\omega$ . Here, the two important parameters are  $n$ , and the EoS parameter,  $k$ . With a slight variation of all other parameters we have considered two extremal variations for  $k$  and  $n$ . When the EoS parameter  $k$  is 1, i.e., stiff perfect fluid, one we get positive solutions for higher values of  $n$ , irrespective of the value of the  $\omega$  in the range  $-4 < \omega < 3$ . In the limit  $r \rightarrow 0$ , i.e. near the singularity, as  $n$  increases  $P(r)$  decreases, thus lowering the value of the Galileon scalar field  $\phi$ . Apparently, it says lesser the effect of the Galileon scalar field upon the potential more is the chance to have a NS. This is contrary to our expectation from our previous experience of gravitational collapse in Brans-Dicke theory [49]. In radiation era ( $k = \frac{1}{3}$ ), naked singularity is the ultimate fate of the universe irrespective of the values of the parameters. In accelerating universe ( $k = -0.5$ ) the trend is almost similar to the radiation era. Except for  $n = 2$ , the collapse always results in a naked singularity. At phantom crossing ( $k = -1$ ) we see that the tendency of getting a naked singularity increases as the value of  $\omega$  is decreased. In phantom era ( $k = -2$ ) contrary to our expectations from our previous experience [49] the tendency of occurrence of naked singularity decreases appreciably. Nevertheless we get naked singularities for very high values of  $n$ , i.e. under the influence of lower values of the Galileon scalar field. As only two parameters are controlling the end fate, we will plot their variations in the fig 1a – o. In figures 1a to 1o we have plotted the  $k - X_0$  contours for increasing values of  $\omega$  and  $n$ .

## 4 Graphical Analysis

Contour plots are generally drawn to show the simultaneous variations of more than one quantity in a 2D plot. In the algebraic eqn. 28, we can see that there are many parameters. Now out of these, the main parameters that really control the collapsing scenario are  $n$  and  $\omega$ , as determined from the tables. Since here we are interested in studying the end state of collapse in different cosmological eras (i.e. for different values of  $k$ ), we have been inclined to generate  $k$  vs  $X_0$  plot. The contour lines show the existence of positive solutions of  $X_0$  for various values of  $k$ , for a particular set of values of the parameters involved. It can be seen that for a particular set of parametric values, eqn. 28 becomes non-linear and hence more than one solution exists. In the figures, each contour lines correspond to one such solution of  $X_0$ . The regions having identical colours exhibit almost identical properties. It is seen that, as the value of  $n$  is increased, the contour lines get raised and crowd around higher values of  $X_0$ . So the dependence on  $n$  is quite clear. For relatively higher values of  $n$ , there is a greater tendency of getting positive roots of  $X_0$ , resulting in a naked singularity.

## 5 Conclusion

In this work, first we have assumed the spherically symmetric space-time model with Vaidya null radiation and perfect fluid in the background of one of the modified gravity like Galileon gravity, which is more generalized form of Brans-Dicke gravity. We have determined the solutions of Einstein equations in Galileon gravity thus constituting the generalized Vaidya metric. The solution may be called generalized Vaidya solution in Galileon gravity theory. Then we investigated the existence of the radial null geodesic from the final fate of the collapsing object. Chances of having such geodesic points corresponds to the chances of having a NS. If not then the construction of a BH is confirmed. In this paper we see that by proper fine tuning of the parameters it is always possible to have a uncensored singularity as the fate of the gravitational collapse in Galileon gravity. We know from our previous works [44, 49] that a strong modified gravity like the Brans-Dicke gravity censors the singularity which is formed by the gravitational collapse. The role of EoS parameter  $k$  has been analyzed for different eras. When the EoS parameter  $k$  is 1, i.e., stiff perfect fluid, we get positive solutions for higher values of  $n$ , irrespective of the value of the  $\omega$  in the range  $-4 < \omega < 3$ , which determines the possibility of NS. For radiation and dark energy eras, the NS is the ultimate fate of the gravitational collapse. At phantom crossing ( $k = -1$ ), we see that the tendency of getting a naked singularity increases as the value of  $\omega$  is decreased. In phantom era ( $k = -2$ ), the tendency of occurrence of naked singularity decreases. In figures 1a to 1o we also see that the natures of  $k - X_0$  contours for varying values of two parameters  $\omega$  and  $n$ . From the plots we get an idea that the possibility of NS increases for higher values of  $n$  and the lower values of  $\omega$ . From this we can conclude that the Galileon gravity, which is an infrared modification of Einstein gravity is perhaps not as strong a gravity theory as its counterparts.

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